Introduction For Instructors

With all projects we create, modify and/or utilize it is advantageous for multiple learning styles to include a three pronged approach: Numerical, Analytical and Graphical. As we become proficient in developing investigations covering these three areas, our students will become more productive at understanding and utilizing mathematics.

There are some topics which by the nature of their complexity, may only lend themselves to one or two these approaches. But, in later courses, one has the opportunity to review algebraic solutions for a problem, review the graphical techniques and then extend student understanding to calculus. A fine example of such a problem is when students are asked to determine the maximum area a gardener may enclose with a given length of fence where one side of the rectangular garden will lie along a garage or other building.

Numerically → Students may simply take random dimensions (based on the given length) and use the area formula for a rectangle to "guess" the greatest area.

Graphically → With an Algebraic background, a student can construct a quadratic representation for the rectangle's area …with the aid of a graphing utility, finding the maximum of a parabola is rather quick and very exact.

Analytically → Once a student has a visual representation of the parabola – a quadratic function's graph – there is a great opportunity to marry the visual (graphical) with the analytical. The maximum area occurs at the vertex of the parabola … which can be found on the graphing utility as well as analytically – the *x*-value of the vertex for a quadratic function in the form $f(x) = ax^2 + bx + c$ is determined by $x = \frac{-b}{2a}$. Since the *x*-value is *where* the maximum occurs, the actual *what* is determined by $f\left(\frac{-b}{2a}\right)$.

With differential calculus, further concepts of algebra & geometry can be molded with the parabolic model connecting a horizontal tangent line (zero slope) to the vertex of the parabola to justify the existence of the maximum area value through a first derivative sign analysis.

The following activity has many merits … Among them, motivation for educational success and determination, a motivation for understanding our PSSA anchors, as well as an example of this three pronged approach to problem solving.

"Education Can Pay Off"

Introduction: The Patriot News offered figures on various educational achievements to average income. With some help from our career counselor, Mr. Harrison, we have been able to add a rapidly growing category for your consideration – Associate Technical Degree. Generally, Associate Technical degrees are two years. However, many trades require an apprentice period prior to earning full wages.

C4 THE PATRIOT-NEWS THURSDAY, OCTOBER 26, 2006

Education can pay off

The Associated Press

How much is a bachelor's degree worth? About $23,000 a year, the government said in a report yesterday.

That is the average gap in earnings between adults with bachelor's degrees and those with high school diplomas, according to data from the Census Bureau.

College graduates earned an average of $51,554 in 2004, the most recent figures available, compared with $28,645 for adults with a high school diploma.

High school dropouts earned an average of $19,169, and those with advanced college degrees made an average of $78,093.

The income gap narrowed slightly from five years earlier, when college graduates made nearly twice as much as high school graduates. But the differences remained significant for men and women of every racial and ethnic group.

Eighty-five percent of people 25 and older had at least a high school diploma or the equivalent in 2005, according to the Census Bureau's 2005 Current Population Survey.

In 2000, 80 percent had a high school diploma or the equivalent, and a little more than half did in 1970.

For comparison sake, we will assume all workers retire at age 62

- Our worker drops out at age 16
- HS workers graduate at age 18
- Associate degree takes 2 years beginning work at age 20
- Bachelor's degree is typically 4 years – work begins at 22 years of age
- Advanced College Degree begins work at age 25.

Also, we will ignore cost of living increases which undoubtedly raise the average income over one's working career. Given these assumptions, use information from the Patriot News article entitled "Education Can Pay Off" to complete the following table. Based on data provided by Mr. Harrison, the average annual earnings of 14 Associate Technical programs is $40,400.

Numerical Approach	High School Dropout	High School Graduate	Associate/ Technical	Bachelor Degree	Advanced College Degree
Average Annual Income	$19,169	$28,645	$40,400	$51,554	$78,093
Working years till retirement @ age 62	46	44	42	40	37
Total Lifetime Earnings	$881,774	$1,260,380	$1,696,800	$2,062,160	$2,889,441

Comparison … Analytical Approach

1. Based on the data, what is the difference in life earnings between a High School Graduate and a High School Dropout?

   $1,260,380 – $881,774 = $378,606

2. As a result of earning an Associate/Technical degree, what change in earning potential results in comparison to a High School graduate?

   $1,696,800 – $1,260,380 = $436,420

3. From the table, how large is the gap in lifetime earning potential between a two year technical degree and a four year collegiate degree?

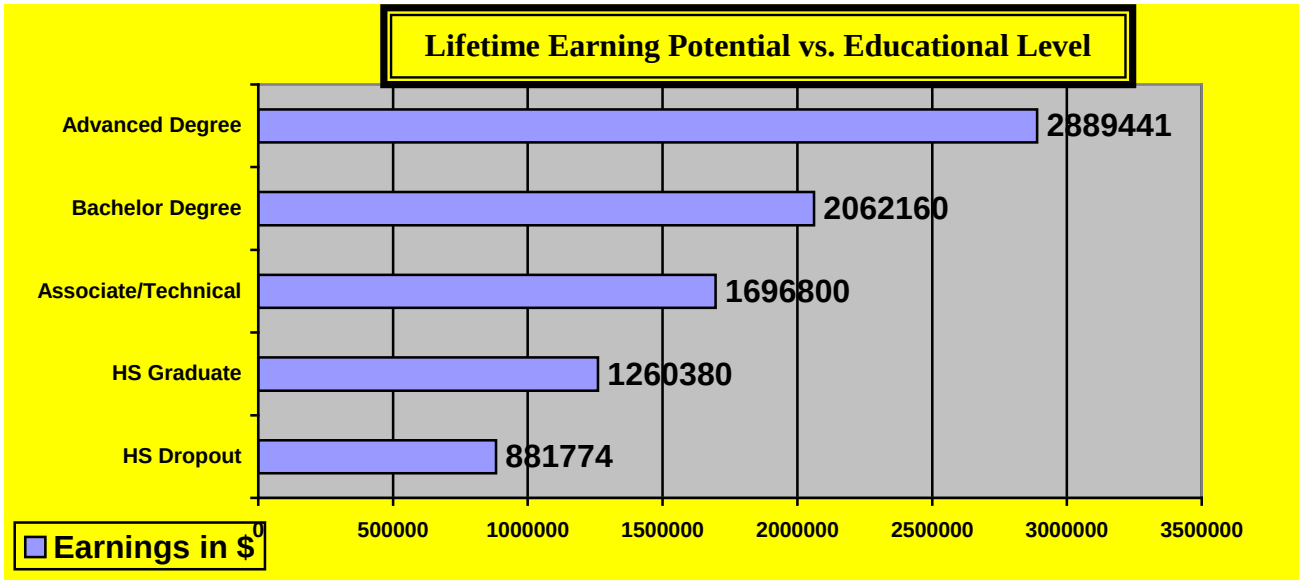
   $2,062,160 – $1,696,800 = $365,360

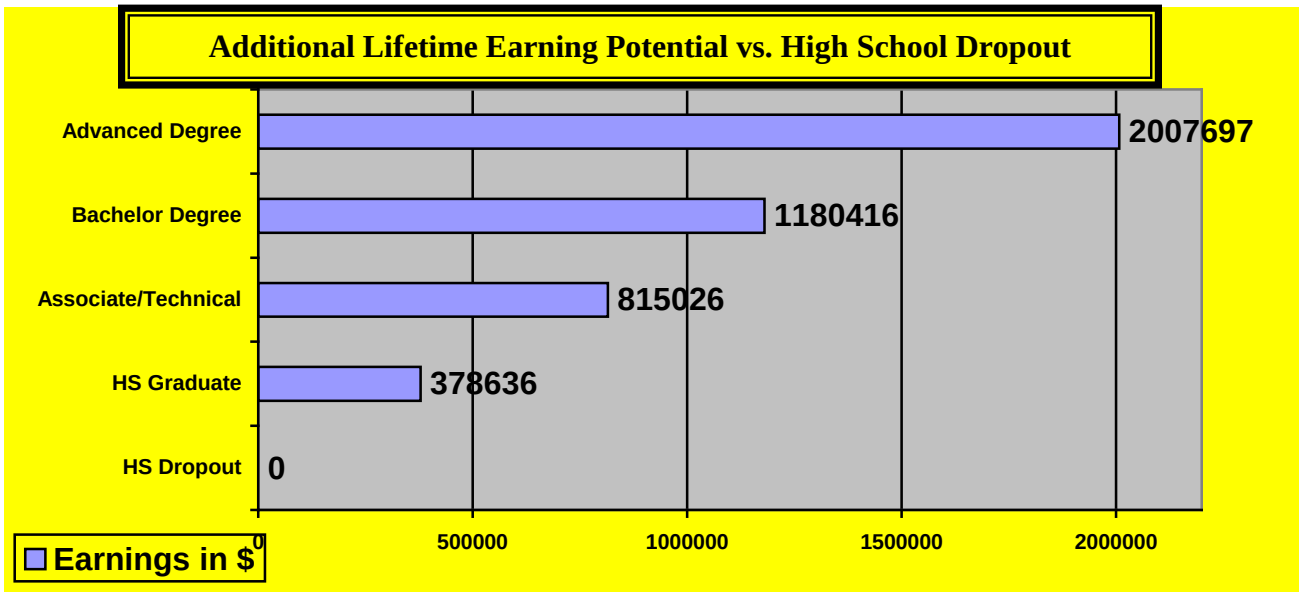
4. Repeat this comparison between the Advanced Degree verses the Bachelor Degree groups.

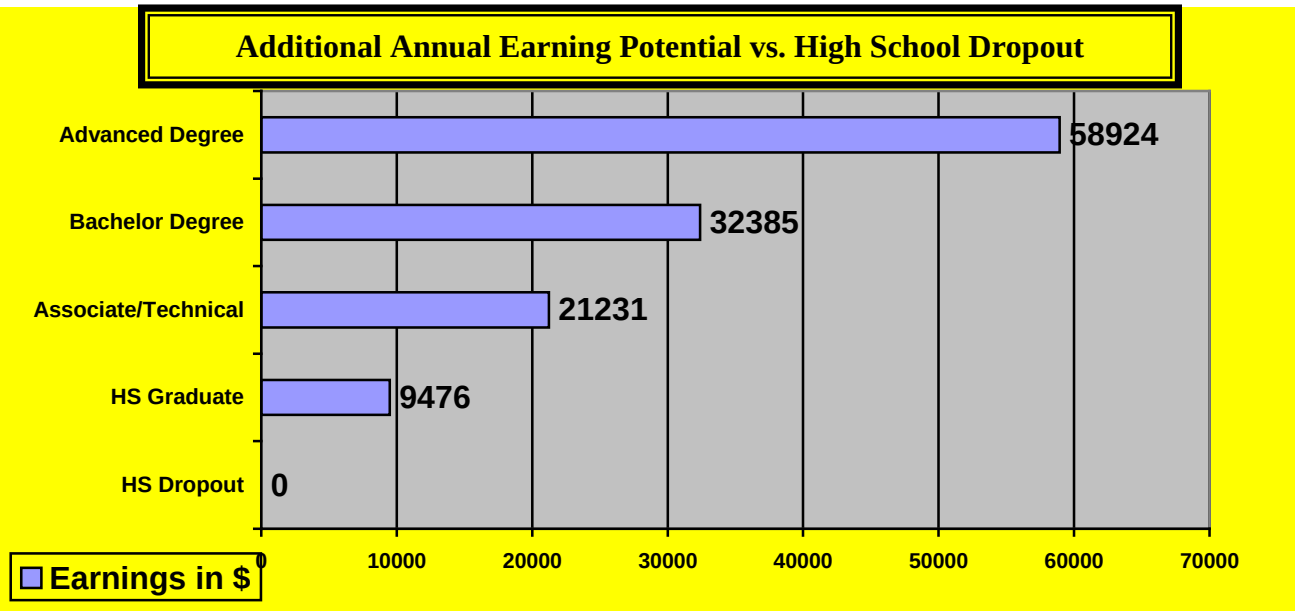
   $2,889,441 – $2,062,160 = $827,281

Comparison … Graphical Approach

We will now construct three separate Bar graphs using our data. First, total earnings of a worker verses educational level achieved; second, additional lifetime earning potential verses a high school dropout; and the final graph is to show additional annual earning potential verses a high school dropout. Be sure each graph has a title as well as appropriate labels. Include with your bars the actual dollar amounts.







Which of these graphical comparisons makes the most impact on your personal outlook toward the value an education plays in terms of earning potential? How has this project changed your outlook on the value of maximizing your educational abilities? Explain using complete sentences.

Answers will vary…