**Introduction For Instructors**

With all projects we create, modify and/or utilize it is advantageous for multiple learning styles to include a three pronged approach: Numerical, Analytical and Graphical. As we become proficient in developing investigations covering these three areas, our students will become more productive at understanding and utilizing mathematics.

There are some topics which by the nature of their complexity, may only lend themselves to one or two these approaches. But, in later courses, one has the opportunity to review algebraic solutions for a problem, review the graphical techniques and then extend student understanding to calculus. A fine example of such a problem is when students are asked to determine the maximum area a gardener may enclose with a given length of fence where one side of the rectangular garden will lie along a garage or other building.

Numerically → Students may simply take random dimensions (based on the given length) and use the area formula for a rectangle to “guess” the greatest area.

Graphically → With an Algebraic background, a student can construct a quadratic representation for the rectangle’s area …with the aid of a graphing utility, finding the maximum of a parabola is rather quick and very exact.

Analytically → Once a student has a visual representation of the parabola – a quadratic function’s graph – there is a great opportunity to marry the visual (graphical) with the analytical. The maximum area occurs at the vertex of the parabola … which can be found on the graphing utility as well as analytically – the x-value for a quadratic function in the form \( f(x) = ax^2 + bx + c \) is determined by \( x = \frac{-b}{2a} \). Since the x-value is *where* the maximum occurs, the actual *what* is determined by \( f\left(\frac{-b}{2a}\right) \).

With differential calculus, further concepts of algebra & geometry can be molded with the parabolic model connecting a horizontal tangent line (zero slope) to the vertex of the parabola to justify the existence of the maximum area value through a first derivative sign analysis.

The following activity has many merits … Among them, motivation for educational success and determination, a motivation for understanding our PSSA anchors, as well as an example of this three pronged approach to problem solving.
“Education Can Pay Off”

Introduction: The Patriot News offered figures on various educational achievements to average income. With some help from our career counselor, Mr. Harrison, we have been able to add a rapidly growing category for your consideration - Associate Technical Degree. Generally, Associate Technical degrees are two years. However, many trades require an apprentice period prior to earning full wages.

For comparison sake, we will assume all workers retire at age 62
- Our worker drops out at age 16
- HS workers graduate at age 18
- Associate degree takes 2 years beginning work at age 20
- Bachelor’s degree is typically 4 years - work begins at 22 years of age
- Advanced College Degree begins work at age 25.

Also, we will ignore cost of living increases which undoubtedly raise the average income over one’s working career. Given these assumptions, use information from the Patriot News article entitled "Education Can Pay Off" to complete the following table. Based on data provided by Mr. Harrison, the average annual earnings of 14 Associate Technical programs is $40,400.
## Numerical Approach

<table>
<thead>
<tr>
<th></th>
<th>High School Dropout</th>
<th>High School Graduate</th>
<th>Associate/Technical</th>
<th>Bachelor Degree</th>
<th>Advanced College Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Annual Income</td>
<td>$19,169</td>
<td>$28,645</td>
<td>$40,400</td>
<td>$51,554</td>
<td>$78,093</td>
</tr>
<tr>
<td>Working years till retirement @ age 62</td>
<td>46</td>
<td>44</td>
<td>42</td>
<td>40</td>
<td>37</td>
</tr>
<tr>
<td>Total Lifetime Earnings</td>
<td>$881,774</td>
<td>$1,260,380</td>
<td>$1,696,800</td>
<td>$2,062,160</td>
<td>$2,889,441</td>
</tr>
</tbody>
</table>

### Comparison … Analytical Approach

1. Based on the data, what is the difference in life earnings between a High School Graduate and a High School Dropout?

   \[\$1,260,380 - \$881,774 = \$378,606\]

2. As a result of earning an Associate/Technical degree, what change in earning potential results in comparison to a High School graduate?

   \[\$1,696,800 - \$1,260,380 = \$436,420\]

3. From the table, how large is the gap in lifetime earning potential between a two year technical degree and a four year collegiate degree?

   \[\$2,062,160 - \$1,696,800 = \$365,360\]

4. Repeat this comparison between the Advanced Degree verses the Bachelor Degree groups.

   \[\$2,889,441 - \$2,062,160 = \$827,281\]
We will now construct three separate Bar graphs using our data. First, total earnings of a worker verses educational level achieved; second, additional lifetime earning potential verses a high school dropout; and the final graph is to show additional annual earning potential verses a high school dropout. Be sure each graph has a title as well as appropriate labels. Include with your bars the actual dollar amounts.

**Comparison … Graphical Approach**

**Lifetime Earning Potential vs. Educational Level**

- Advanced Degree: $2889441
- Bachelor Degree: $2062160
- Associate/Technical: $1696800
- HS Graduate: $1260380
- HS Dropout: $881774

**Additional Lifetime Earning Potential vs. High School Dropout**

- Advanced Degree: $2007697
- Bachelor Degree: $1180416
- Associate/Technical: $815026
- HS Graduate: $378636
- HS Dropout: $0
Which of these graphical comparisons makes the most impact on your personal outlook toward the value an education plays in terms of earning potential? How has this project changed your outlook on the value of maximizing your educational abilities? Explain using complete sentences.

*Answers will vary...*