**Rule of 72**

What is it?  How does it work?

The Rule of 72 is a tool used by investors to approximate doubling time of an investment. But, how does it work? The amount in an investment for $t$ years can be modeled by the formula $A(t) = P \left(1 + \frac{r}{n}\right)^n$. In this formula $A(t)$ is the amount after time $t$, $P$ is the principle, $r$ is the interest rate as a decimal, $n$ is the number of times compounded annually and $t$ is time in years.

To begin our investigation into the Rule of 72, we will assume an initial investment of $1000, set some interest percentages and look for the time it would take the investment to double. To help standardize our investigation, we will set our compounding to annual. This will reduce our working equation for the amount of money in the investment after time $t$ to $A(t) = P(1+r)^t$. In order to use our graphing utilities, we will modify this equation to the form $y = 1000(1+r)^x$. In this version, the variable $x$ is the total amount in the account and variable $x$ represents time of the investment in ________.

**Comparison ... Numerical Approach - 2nd Table (of Values)**

In the $y =$ mode of your graphing utility, enter four separate equations - one each for 3%, 4%, 8% and 12% interest rates. Record your equations below.

\[
\begin{align*}
    y_1 &= \text{________________} \\
    y_2 &= \text{________________} \\
    y_3 &= \text{________________} \\
    y_4 &= \text{________________}
\end{align*}
\]

Rounded to the nearest year, record the time ($x$-value) it would take each investment type to double ... reach a ______-value of ________. The table feature of your graphing utility will help.

<table>
<thead>
<tr>
<th>Doubling Time of an Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>12</td>
</tr>
</tbody>
</table>
Comparison ... Graphical Approach Option #1 \(\rightarrow\) 2nd Calc 1: value

A second approach to this problem can be pursued from the graph screen of your graphing utility. The table investigation above will help us to set our window values for this investigation. Since \(x\) represents _____ and \(y\) represents the amount of __________ in our account, the \(x_{\text{min}}\) can begin at zero while the \(x_{\text{max}}\) should be approximately __________ with \(x_{\text{scl}}\) of 2. The \(y_{\text{min}}\) should also be __________ (our initial investment) with a \(y_{\text{max}}\) of approximately __________ and \(y_{\text{scl}}\) of 100.

Now, with the 2nd Calc 1: value option on your graphing utility, we can begin to guess \(x\)-values (_______) that may cause our investment(s) to reach a value of $2000 - or ________. What might help us be more efficient in this approach?

Add a fifth line – a horizontal line representing our target doubling amount… Generally __________________ or specifically ____________________.

This fifth ________________ line, which we can bold at the \(y = \) screen will help our efficiency in estimating the number of years required for each investment to double.

Does anyone know a faster method?

Comparison ... Graphical Approach Option #2 \(\rightarrow\) 2nd Calc 5:intersect

Using the intersect feature on our graphing utility, we can see how to quickly find the doubling values …

The intersection point is given in the form \((x, y)\), what are the meanings of each part of this ordered pair? \((____________________ , ______________________)\)
Comparison ... Analytical Approach

Using logarithms, this exponential equation can be solved analytically...

What does all this mean???

Since as the interest rate increases the time it takes our investment to double
___________ - the Rule of 72 is an __________ relationship ... that is, The Rule of
72 follows the pattern, model, of __________.  
Now, for our final investigative step ... in an inverse model, \( y = \frac{k}{x} \),

___________ is inversely proportional to ____________ ...
translates to "people speak" in this problem as ...

___________ is inversely proportional to ____________ ...

In equation format we have  \( \text{Years to Double} = \frac{k}{\text{Interest Rate}} \)

By inserting a set of data from our table into this inverse variation model ... say 8%
takes ______ years to double ... we obtain

Thus, the Rule of 72 is finalized as __________ Time of an Investment = \( \frac{72}{\text{Rate}} \).

And, the purpose of the Rule of 72 is to ___________ the number of years it
would take for an investment to ______ at a particular interest _________. To
approximate doubling time, in years, we simply ________ 72 by a rate of interest.