Rule of 72

What is it? How does it work?

The Rule of 72 is a tool used by investors to approximate doubling time of an investment. But, how does it work? The amount in an investment for t years can be modeled by the formula $A(t)=P\left(1+\frac{r}{n}\right)^{nt}$. In this formula $A(t)$ is the amount after time t, P is the principle, r is the interest rate as a decimal, n is the number of times compounded annually and t is time in years.

To begin our investigation into the Rule of 72, we will assume an initial investment of $1000, set some interest percentages and look for the time it would take the investment to double. To help standardize our investigation, we will set our compounding to annual. This will reduce our working equation for the amount of money in the investment after time t to $A(t)=P(1+r)^{t}$. In order to use our graphing utilities, we will modify this equation to the form $y=1000(1+r)^{x}$. In this version, the variable y is the total amount in the account and variable x represents time of the investment in years.

Comparison … Numerical Approach – 2nd Table (of Values)

In the $y=$ mode of your graphing utility, enter four separate equations – one each for 3%, 4%, 8% and 12% interest rates. Record you equations below.

$y_1=1000(1.03)^{x}$ $y_2=1000(1.04)^{x}$

$y_3=1000(1.08)^{x}$ $y_4=1000(1.12)^{x}$

Rounded to the nearest year, record the time (x-value) it would take each investment type to double … reach a y-value of $2000. The ***table*** feature of your graphing utility will help.

Doubling Time of an Investment	
Percentage	Doubling Time
3	23 or 24
4	18
8	9
12	6

Comparison … Graphical Approach Option #1 → 2nd Calc 1:value

A second approach to this problem can be pursued from the ***graph*** screen of your graphing utility. The table investigation above will help us to set our window values for this investigation. Since x represents time and y represents the amount of money in our account, the **xmin** can begin at zero while the **xmax** should be approximately 30 with ***xscl*** of 2. The **ymin** should also be 1000 (our initial investment) with a **ymax** of approximately 3000 and **yscl** of 100.

Now, with the **2nd Calc 1:value** option on your graphing utility, we can begin to guess x-values (time) that may cause our investment(s) to reach a value of $2000 – or double. What might help us be more efficient in this approach?

Add a fifth line – a horizontal line representing our target doubling amount … this offers an opportunity to discuss horizontal lines are of the form $\underline{y_5 = \text{constant}}$ or specifically $\underline{y_5 = 2000}$.

This fifth horizontal line, which we can bold at the $y =$ screen will help our efficiency in estimating the number of years required for each investment to double.

Does anyone know a faster method?

Comparison … Graphical Approach Option #2 → 2nd Calc 5:intersect

Using the intersect feature on our graphing utility, we can show students how to quickly find the doubling values … this reinforces algebraic methods for finding solutions to systems of equations (analytical methods) despite the fact that precalculus level algebra skills are required. There is also an opportunity to teach students how to use the "toggle" switch in the $y =$ screen to turn graphs *on* and *off*. The intersection value is given as an ordered pair (x, y) - another great teaching moment …

The intersection point is given in the form (x, y), what are the meanings of each part of this ordered pair? $\left(\text{time of investment in years}, \text{value of investment in \$}\right)$

Comparison … Analytical Approach

Using logarithms, this exponential equation can be solved analytically…

$$A(t) = P(1+r)^t$$
$$2000 = 1000(1+.03)^t$$
$$\frac{2000}{1000} = \frac{\cancel{1000}(1.03)^t}{\cancel{1000}}$$
$$2 = 1.03^t$$
$$\ln 2 = \ln 1.03^t$$

→

$$\ln 2 = \ln 1.03^t$$
$$\ln 2 = t \ln 1.03$$
$$\frac{\ln 2}{\ln 1.03} = \frac{t\,\cancel{\ln 1.03}}{\cancel{\ln 1.03}}$$
$$t = \frac{\ln 2}{\ln 1.03} \approx 23.44977$$

What does all this mean???

Since as the interest rate increases the time it takes our investment to double decreases – the Rule of 72 is an inverse relationship … that is, The Rule of 72 follows the pattern, model, of $y = \frac{k}{x}$.

Now, for our final investigative step … in an inverse model, $y = \frac{k}{x}$,

y is inversely proportional to x …
translates to "people speak" in this problem as …
years to double is inversely proportional to **rate of investment** …

In equation format we have $\text{Years to Double} = \frac{k}{\text{Interest Rate}}$

By inserting a set of data from our table into this inverse variation model … say 8% takes 9 years to double … we obtain

$$\text{Years to Double} = \frac{k}{\text{Interest Rate}}$$
$$9 \cdot 8 = \frac{k}{\cancel{8}} \cdot \cancel{8}$$
$$72 = k$$

Thus, the **Rule of 72** is finalized as $\text{Doubling Time of an Investment} = \frac{72}{\text{Interest Rate}}$.

And, the purpose of the **Rule of 72** is to approximate the number of years it would take for an investment to double at a particular interest rate. To approximate doubling time, in years, we simply divide 72 by a rate of interest.